

MATHEMATICAL MODELLING FOR ECOSYSTEM RESTORATION AND SUSTAINABLE MANAGEMENT OF BUSHBUCKRIDGE NATURE RESERVE

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Abstract

In this report, we develop a mathematical model for the revitalization of the ecosystem in Bushbuckridge Nature Reserve, Mpumalanga, South Africa, as part of ongoing restoration efforts. The primary objectives are to restore biodiversity, address human-wildlife conflict, and optimize resource allocation within the park. We present a comprehensive modeling framework incorporating ecological dynamics, human intervention, and spatial optimization to achieve this. We introduce an optimization-based space allocation model to strategically distribute wildlife and physical infrastructure strategically, ensuring ecological balance and sustainable tourism. The results suggest that herbivores and vegetation resources should be positioned near park edges for community benefit, while predators should be placed centrally to maintain ecological stability. Our findings highlight that a collaborative, data-driven approach is essential for the park's sustainability.

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1 Introduction

Tourism is a major contributor to South Africa's economy. Malik-Nair (2023) stated that “travel and tourism contributes 8.6% to the South African economy and provides for 9.2% of total employment within the country”. Undoubtedly, tourist attractions such as nature reserves play an essential role in this sector, as South Africa is a global leader in wildlife tourism. However, some government-owned protected sites, such as Bushbuckridge Nature Reserve (BNR), have struggled to keep up economically while dealing with the local population's needs. Barnes and Pearce (1992) believe that “wildlife tourism offers both a major economic potential for developing countries, even allowing for the necessary investment in infrastructure, and an opportunity to manage sustainably wildlife resources”.

It is pleasing that BNR is subscribing to this notion, as it has recently set itself an objective of becoming a world-class Nature Reserve that maintains the integrity of its biodiversity through conservation while permitting sustainable use of its natural resources by neighbouring communities (Bushbuckridge Nature Reserve Management Plan, 2016-2022). The Bushbuckridge Nature Reserve is geographically located in the north of Mpumalanga Province, under the Bushbuckridge Local Municipality, see Figure 1. It is surrounded by local communities. It is about 7000 hectares, co-managed by the Mpumalanga Tourism and Parks Agency (MTPA) and Communal Property Associations (CPA) through an Integrated Management Plan. Interestingly, BNR is 100% land claimed by two CPAs, the Sisonke CPA (95%) and Injaka Waterval (5%).

This Bushbuckridge nature reserve presents an opportunity to restore and conserve the ecosystem because it used to be one of the province's well-operational and well-preserved natural reserves. Possible partnerships with local communities that honour their traditions and values, including discussions around human-wildlife conflict, become paramount. Creating employment in restoration and conservation while integrating scientific activities and fully reopening doors to tourists can be considered a significant contributor to the Tourism Sector as part of South Africa's tourism Gross Domestic Production (GDP) from this region.

Management has commendable plans to offer visitors and future generations a safe and unique nature experience at its best [7]. Such plans will ensure a high quality of life for BNR. Braithwaite and Reynolds [3] note that high-quality wildlife experiences are arguably at the pinnacle of the natural world experience. On the other hand, one cannot deny that wildlife tourism is about increasing the probability of marvelous encounters with wildlife for visitors whilst protecting wildlife resources [3]. This submission suggested the importance of mathematical models in assisting with challenges faced by protected sites such as BNR existing in the space of biodiversity. Biodiversity is described by Noss [8] as the composition, structure, and function inferred at four levels of organization (regional-landscape), community-ecosystem, population-species, and genetic. Therefore, through management authorities such as the Mpumalanga Tourism and Parks Agency (MTPA), South Africa has recognized the importance of biodiversity protection through legislation. Hence, effective management of declared protected areas must consider biodiversity conservation.

Areas adjacent to the Reserve have undergone habitat losses, and the size and densities of human settlements have increased in Bushbuckridge [4]. The activities of impoverished

communities are seen as a major driver of landscape change. The increases seen in human-impacted vegetation suggest that rural settlements depend on adjacent communal lands for cattle grazing, fuel wood, and medicinal plants. These further stress the importance of the nature reserve as a conservation area and the need for detailed terrestrial biodiversity data.

In addition, Bushbuckridge Nature Reserve has the potential to contribute to conservation as mandated by the National Biodiversity Act (Act No. 10 of 2004) [7] and the Mpumalanga Nature Conservation (Act No. 10 of 1998) [6], and the Reserve's value as a significant catchment area, its position in the Kruger to Canyons Biosphere Reserve, as an ecological corridor should be viewed as an added advantage. The envisaged activities in the nature reserve will link the Blyde River Canyon Nature Reserve and the Kruger National Park in order to maintain and improve the movement of animals and plants through reserves and potentially improve the resilience of ecosystems to climate change.

On the other hand, as is experienced in other nature reserves, human-wildlife conflict and co-existence are serious challenges. This is not an exception for the Bushbuckridge Nature Reserve, which includes livestock from communities residing on the boundary that enters the Reserve since some of the fence is down. Nyhus [9] believes that human interactions with wildlife are a defining experience of human existence, although these interactions can be positive or negative. It is understood that people compete with wildlife for food and resources and have eradicated dangerous species, co-opted and domesticated valuable species, and applied a wide range of social, behavioral, and technical approaches to reduce negative interactions with wildlife [9]. If such conflicts are not addressed, unfortunately, they eventually lead to the extinction and reduction of numerous species and uncountable human deaths and economic losses.



Figure 1: Bushbuckridge Nature Reserve surface map.

In this manuscript, we develop a mathematical model to determine what can be done

to revive the ecosystem of Bushbuckridge Nature Reserve, since there is currently no life or meaningful activity there. The proposed approach integrates ecological, environmental, and socio-economic components. These are meant to address and satisfy the needs of all involved stakeholders who might be affected by the revitalization of the nature reserve. As such, we narrow the objectives and scope to reviving the ecosystem of the reserve to restore biodiversity and ecological functionality with our focus on species reintroduction, vegetation recovery, soil rehabilitation, and water management.

The remainder of the manuscript is organized as follows: In Section 3, we present a mathematical model that describe the revitalization process of the reserve. The models are further analyzed for their meaningfulness and well-posedness both biologically and mathematically. In Section 4, we provide some basic simulations of the proposed models and their results. We interpret these Results and provide a meaning analysis for all stakeholders to easily understand. Lastly, in Section 5, we provide key findings of the research work and provide a clear way forward that all stakeholders may take to ensure the proper revitalization of this prestigious tourist attraction.

2 Problem statement

Bushbuckridge Nature Reserve is at a pivot point: ecological activity is low, boundary infrastructure is compromised, and tourism value is under-realised, yet the reserve sits in a strategic corridor between Blyde River Canyon Nature Reserve and Kruger National Park. Adjacent communal lands depend on grazing, fuelwood and medicinal plants, intensifying pressure at the park-people interface. Fence failures permit two-way incursions, livestock into the reserve and wildlife into community areas-raising human-wildlife conflict, eroding biodiversity, lowering visitor safety and satisfaction, and weakening local support for conservation.

Within this context, management must select an implementable initial spatial layout across forty-one zones. A finite set of facilities - wildlife species/groups, habitat features (trees, lakes), and built structures (offices, accommodation, dams). The facilities must be positioned so that the plan maximizes the visibility and access to safe wildlife for visitors while preserving ecological integrity and reducing conflict near the boundaries. The layout must respect non-negotiables: predators kept at safe distances from accommodation and offices and away from boundary zones; incompatible species not co-located; species with strong habitat needs placed only in feasible zones (e.g., hippo/crocodile with permanent water); and per-zone capacity and compatibility limits that prevent overcrowding and unsafe co-occurrence.

To support this decision, we formulate a facility layout optimisation model that assigns facilities to zones using visibility scores derived from routes/viewpoints and feasibility masks reflecting ecological and safety rules. Distance-based prohibitions enforce predator–infrastructure separation; boundary rules curb high-risk placements; and capacity constraints limit per-zone load by class (wildlife, vegetation, structures). The objective is to maximise total safe visibility subject to these constraints, producing a practical baseline layout that can be implemented within current budgets and used to guide phased restoration, conflict mitigation and the re-opening of tourism. Success is measured by improved visitor viewing opportunities, reduced

predicted conflict exposure along boundary segments, and ecologically sensible starting configurations for priority species and habitats.

3 Facility layout design model

In this section, we develop a space allocation model to optimally distribute both wildlife and physical infrastructure, such as park offices, dams, and accommodation facilities, within the nature reserve. The objective is to create a facility allocation model that assists management in designing an optimal initial layout for the nature reserve. Facilities should not be placed arbitrarily; instead, they must be strategically positioned to maximize the tourist experience while ensuring ecological sustainability and minimizing human-wildlife conflicts. All physical structures should be easily accessible, and animals should be placed in locations where they are visible to visitors to enhance their experience. Additionally, wildlife placement should support a self-sustaining ecosystem for as long as possible.

The problem of restoring an ecosystem within a nature reserve is very broad and extensive. Therefore, we narrow the focus to the optimal allocation of resources within the nature reserve. Using Buchbuckridge as the case study, the nature reserve is zoned into 41 distinct locations, as determined by the reserve management, see Figure. 2. Our goal is to allocate facilities such as dams, offices, accommodation, vegetation, and wildlife optimally within the reserve. The items to be allocated are collectively referred to as "facilities" and are summarized in Table 1.

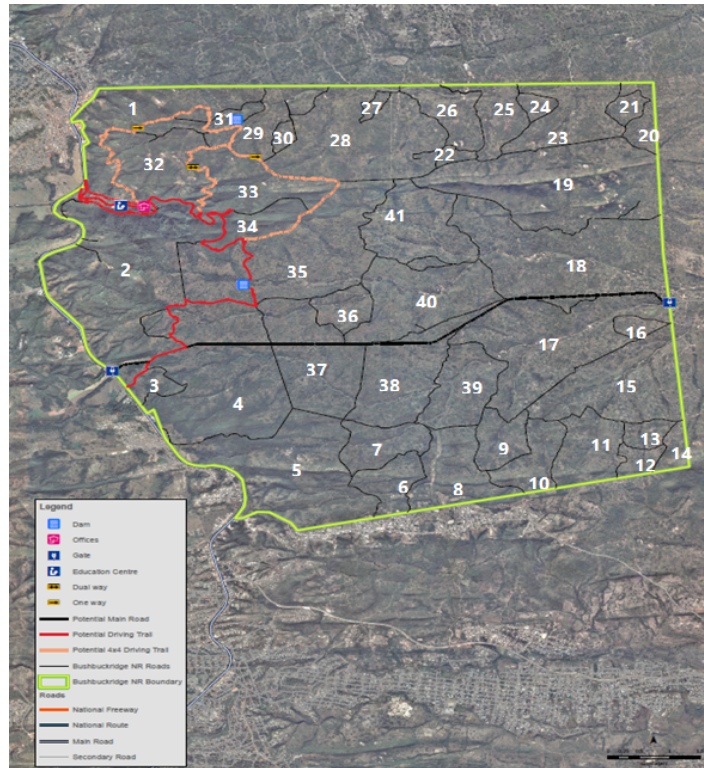


Figure 2: Zonation of Bushbuckridge Nature Reserve.

Number	Facility	Number	Facility	Number	Facility
1	Lion	9	Wild dogs	17	Hyena
2	Rhino	10	Guinea fowls	18	Ostrich
3	Cheetah	11	Crocodile	19	Baboon
4	Elephant	12	Buck	20	Lakes
5	Buffalo	13	Snakes	21	Tall trees
6	Hippopotamus	14	Birds	22	Thorn trees
7	Giraffe	15	Leopard	23	Buildings
8	Zebra	16	Warthog	24	Fruit trees

Table 1: Facilities to be distributed in the nature reserve.

The initial layout model for Bushbuckridge Nature Reserve is formulated as an optimization problem with the following objectives: (i) to place animals in locations where they can best adapt, and (ii) to enhance visitor experience by maximizing the visibility of wildlife. A binary decision variable X_{ij} is introduced, where $X_{ij} = 1$ if facility i is assigned to location j , and 0 otherwise. The objective function aims to maximize both adaptability and visibility and is mathematically defined as:

$$\max_X \sum_{i \in I} \sum_{j \in J} \sum_{k \in I} \sum_{l \in J} d_{ij} f_{kl} X_{ij} X_{kl},$$

where d_{ij} represents the adaptability of facility i at location j , and f_{kl} denotes the visibility factor of facility i at location j , relative to designated tourist trails. The sets I and J correspond to facilities and locations, respectively.

The objective function is subject to several constraints to ensure ecological balance, visitor safety, and sustainable resource use. The constraints are:

1. Each facility must be assigned to exactly one location:

$$\sum_{j \in J} X_{ij} = 1, \quad \forall i.$$

2. To prevent overcrowding, each location can host a maximum of two facilities:

$$\sum_{i \in I} X_{ij} \leq 2, \quad \forall j.$$

3. Offices and accommodation facilities must be placed at a safe distance from predators:

$$\sum_{i \in I_P} X_{ij'} \leq 1 - X_{oj}, \quad \forall j \in J, j' \in J_j.$$

where I_P is the set of predators, J_j represents locations adjacent to j , and X_{oj} is the variable for office placement.

4. Vegetation must be allocated near the boundaries to provide resources such as firewood and medicinal plants for the local community:

$$\sum_{j \in J_B} X_{Trees,j} \geq 1,$$

where J_B denotes locations on the boundary of the nature reserve.

5. Herbivores should be placed near boundary locations to allow for controlled hunting:

$$\sum_{i \in I_V} \sum_{j \in J_B} X_{ij} \geq 1.$$

where I_V represents prey species.

6. Following the above constraint, predators cannot be placed near boundary locations:

$$\sum_{i \in I_P} \sum_{j \in J_B} X_{ij} = 0.$$

7. Predators and prey cannot be placed in the same location to ensure ecosystem sustainability:

$$X_{kj} + X_{lj} \leq 1, \quad k \in I_V, \quad l \in I_P, \quad \forall j.$$

This will avoid the quick consumption of prey by predators after reintroduction to the nature reserve.

8. Certain animal species, such as cheetahs and baboons, cannot cohabit due to natural conflicts:

$$X_{3,j} + X_{19,j} \leq 1, \quad \forall j,$$

where $X_{3,j}$ is the variable for allocations cheetahs at location j and $X_{19,j}$ is the variable for allocating baboons at location j .

Given that solving a quadratic optimization problem can be complex, we simplify the model by assuming that all animals can adapt to any location in the nature reserve. Thus, we set the adaptability factor $d_{ij} = 1$. Additionally, we assume that the terrain is suitable for all facilities. As a result, the problem simplifies to a linear optimization model that maximizes

facility visibility, enhancing visitor experience. The final mathematical formulation is:

$$\max_X \sum_{i \in I} \sum_{j \in J} f_{ij} X_{ij}, \quad (1a)$$

$$\text{subject to: } \sum_{j \in J} X_{ij} = 1, \quad \forall i, \quad (1b)$$

$$\sum_{i \in I} X_{ij} \leq 2, \quad \forall j, \quad (1c)$$

$$\sum_{i \in I_P} X_{ij'} \leq 1 - X_{oj}, \quad \forall j \in J, j' \in J_j, \quad (1d)$$

$$\sum_{j \in J_B} X_{Trees,j} \geq 1, \quad (1e)$$

$$\sum_{i \in I_v} \sum_{j \in J_B} X_{ij} \geq 1, \quad (1f)$$

$$\sum_{i \in I_P} \sum_{j \in J_B} X_{ij} = 0, \quad (1g)$$

$$X_{3,j} + X_{19,j} \leq 1, \quad \forall j. \quad (1h)$$

This is a binary linear optimization problem of moderate size. The model can be solved with a standard optimization solver.

4 Results

4.1 Layout strategies

For space allocation, we solve the optimization model (1), which is designed for the Bushbuckridge Nature Reserve. The reserve contains 24 facilities, including both animals and physical structures, that must be optimally distributed across 41 distinct locations. The problem is formulated as a binary linear programming (BLP) model, where each facility is assigned to one location while adhering to the imposed constraints. The visibility factor, f_{ij} , is defined as the ratio of the facility size to its distance from the predefined tourist path. The main focus of this allocation is with reference to the red trail, as illustrated in Figure 2. This ensures that visitors have an optimal experience while also considering the ecological balance of the Nature Reserve.

To solve the space allocation optimization model (1), we use the MATLAB mixed-integer linear programming solver "intlinprog." This package is particularly effective for solving binary optimization problems due to its ability to handle binary variables efficiently. It employs advanced branch-and-bound techniques, making it well-suited for solving large-scale optimization problems. The results obtained from solving the model are presented in Figure 3 and Figure 4.

Figure 3 provides a matrix representation of the solution to model (1). This solution is represented as a sparse binary matrix of dimensions 24×41 . Here, the rows correspond to

different facilities, while the columns represent distinct locations within the reserve. Each entry in the matrix is either 1 or 0. A value of 1 in position (i, j) indicates that facility i is assigned to location j . A value of 0 signifies that facility i is not assigned to location j . From the matrix representation, we can confirm that the solution adheres to the constraints of the optimization model. Each row contains exactly one 1, ensuring that each facility is assigned to only one location. Each column contains at most two 1's, meaning that a location can accommodate at most two facilities. Some columns contain only 0's, indicating that certain locations remain unoccupied, which is expected given that the number of facilities is less than the total number of locations. These results highlight the strategic placement of facilities. Locations with multiple assigned facilities (columns with multiple ones) can be interpreted as key management or tourist hubs, whereas unoccupied locations may be preserved for ecological balance and conservation.

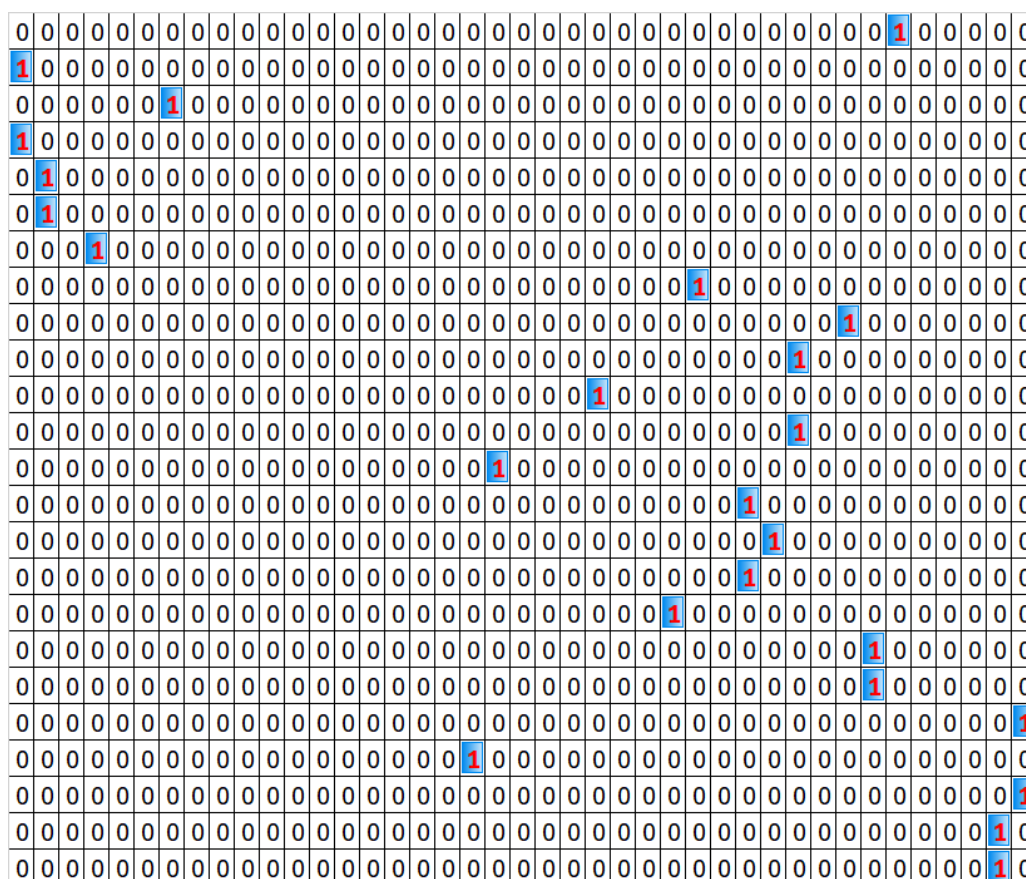


Figure 3: Distribution of facilities in a location grid.

Figure 4 offers a spatial distribution map of the Bushbuckridge Nature Reserve, illustrating the arrangement of wildlife species, vegetation, water bodies, and man-made facilities across the 41 zonation locations. By linking the binary matrix in Figure 3 to the spatial map in Figure 4, we gain insight into how facilities are positioned to align with ecological constraints and management strategies. We observe how facilities are strategically placed based on environmental



Figure 4: Distribution of facilities in the nature reserve.

needs as described by the restrictions imposed on the model.

The lake is centrally located, surrounded by fruit trees and tall trees. This positioning ensures easy access to water while maintaining ecological diversity. Predators and prey are strategically separated following the model constraints to minimize immediate predation. Tourist facilities such as accommodation and park offices are placed in accessible locations away from high-risk predator zones. Vegetation distribution follows the constraints imposed in the model, ensuring that essential plants and trees are placed near the boundaries where they can support the local community by providing resources like firewood and medicinal plants.

From a management perspective, this spatial distribution allows for a critical assessment of the facility placements. High-density areas may serve as focal points for tourism and conservation activities, whereas underutilized locations may present opportunities for further development or ecological preservation. If certain high-biodiversity areas are overly developed, it may indicate a risk of ecosystem strain. Conversely, underutilized zones may indicate untapped opportunities for sustainable tourism and conservation.

5 Conclusions

In this report, we developed a mathematical model for revitalizing the ecosystem in Bushbuckridge Nature Reserve in Mpumalanga, South Africa. The once-abuzz nature reserve is undergoing renovations, and the park management is looking for strategies to bring wildlife

back into the park. The management's main objective is to restore and conserve the ecosystem. They also aim to address human-wildlife conflict through innovative solutions. To address these objectives, we have presented a mathematical model with viable tools for controlling the ecosystem while addressing human-wildlife conflicts.

We used optimization techniques to develop a space allocation model to distribute animals and physical structures in the park optimally. Park management may utilize this model to distribute available facilities to strategically maximize the impact of tourism. Results indicate that herbivores and vegetation (medicinal plants and firewood) must be distributed along the edges of the park to provide maximum impact on the local surrounding communities. Also, the model results indicate the predators must be located more toward the park's center, slightly away from where prey is concentrated.

The revitalization of Bushbuckridge Nature Reserve requires an integrated approach with contributions from all stakeholders. Collaboration with the surrounding communities and scientific institutions around the country is required to ensure the Reserve's smooth running and sustainability. As observed in the study, mathematical models and strategic plans will ensure long-term success in the Bushbuckridge Nature Reserve. Future work and possible extensions of the presented models include applying real data from Bushbuckridge to the population growth and movement models. Also, we can investigate the optimal solution for the non-linear facility design model, incorporating realistic parameters.

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